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BIANCHI TYPE-I BULK VISCOUS COSMOLOGICAL MODELS WITH DECAYING COSMOLOGICAL TERM

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ABSTRACT:

Bianchi type I cosmological models with varying cosmological term Λ and bulk viscous fluid are investigated. Exact solutions of Einstein's field equations have been studied by assuming the law for variation of Hubble's parameter that yields a constant value of deceleration parameter. Physical and kinematical properties of the models are also discussed.

KEYWORDS: *cosmological term Λ, Deceleration parameter q, Hubble parameter H, Scale factor S.*

1. INTRODUCTION

The homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological models, which are used to describe standard cosmological models, are particular case of Bianchi type I, V and IX Universes, according to whether the constant curvature of the physical three-space, t = constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large scale structure of the Universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. In Einstein's theory of general relativity, to account for such an expansion, one needs to introduce some new energy density with a large negative pressure in the present universe, in addition to the usual relativistic or non-relativistic matter. This exotic matter causing cosmic acceleration is known as dark energy. The nature of dark energy is unknown and many radically different models related to this dark energy have been proposed [1, 2].

The simplest explanation of dark energy is provided by the cosmological term Λ, but it needs to be severely fine-tuned due to the problem associated with its energy scale. The vacuum energy density considered today falls below the value of the vacuum energy density predicted by quantum field theory by many order of magnitude [3]. To explain the decay of the density, a number of dynamical models have been investigated in which cosmological term Λ varies with cosmic time t. These models give rise to an effective cosmological term which as long as the universe expands, decays from a large value at initial moment to the small value observed at present. Cosmological models with different decay laws for the variation of cosmological term were studied during last two decades [4-8].

In the investigation of most of the cosmological models, the source of the gravitational field is assumed to be a perfect fluid. But these models do not explain satisfactorily the early stages of evolution. Viscosity may be important in cosmology for a number of reasons. Dissipative mechanisms responsible for smoothing out initial isotropies and the observed high entropy per baryon in the present state of the universe can be explained by involving some kind of dissipative mechanisms e.g. bulk viscosity [9, 10]. Dissipative effects including bulk viscosity are supposed to play a very important role in the early evolution of the universe. During the neutrino decoupling stage, apart from streaming neutrinos moving with fundamental velocity, there is a part behaving like a viscous fluid co-moving with matter. Decoupling of radiation and matter during the recombination era is also expected to give rise to viscous effects. Moreover, a combination of cosmic fluid with bulk dissipative pressure can generate accelerated expansion [11]. Influence of viscosity on the nature of the initial singularity and on the formation of galaxies have been investigated [11, 12]. It has been shown that the coincidence problem can be solved by taking viscous effects into account [13, 14]. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabris et al. [15]. Santos et al. [16] were studied exact solution with bulk viscosity by considering the bulk viscous coefficient as power function of mass density. Johri and Sudarshan [17] were cosidered the effect of bulk viscosity on the evolution of Friedmann models. Cosmological models with bulk viscosity have also been studied by number of author namely, Burd and Coley [18], Maartens [19], Pavon and Zimdahl [20], Pavon et al. [21].

In this paper, we investigate Bianchi type-I cosmological models for bulk viscous fluid distribution with decaying cosmological term Λ. In general relativity we obtain exact solutions of Einstein field equations assuming a law of variation for Hubble's parameter. In Bianchi type-I space time which yields a constant value of deceleration parameter. The law of variation explicitly determines the scale factors. Physical behaviors of the models have been studied.

2. METRIC AND FIELD EQUATION

The spatially homogeneous and anisotropic Bianchi type-I space time is described by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dt^2 \tag{1}$$

The Einstein's field equations with time dependent Λ are

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j + \Lambda g_i^j \tag{2}$$

where energy momentum tensor T_i^j in the presence of bulk viscosity is taken in the form

$$T_i^j = (\rho + \bar{p})v_i v_j + \bar{p}g_i^j \tag{3}$$

where

$$\bar{p} = p - \xi\theta. \tag{4}$$

We assume that the matter content obeys an equation of state

$$p=\omega\rho, \quad 0\le\omega\le 1. \tag{5}$$

Here ρ, p, ξ and θ are the energy density, isotropic pressure, bulk viscosity and expansion scalar respectively. The flow vector v^i satisfies the condition

$$v_i v^i = -1. \tag{6}$$

In co-moving system of coordinates

$$T_1^1 = \bar{p},\ \ T_2^2 = \bar{p},\ T_3^3 = \bar{p},\ \ T_4^4 = \rho.$$

Then field equations are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + \Lambda, \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + \Lambda, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + \Lambda, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\rho+\Lambda\,. \qquad (10)$$

From equations (7)-(10), we obtained

$$\dot{\rho}+(\rho+\overline{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\Lambda}=0. \qquad (11)$$

Eliminating $\overline{p}$ and Λ from (7)-(9) and integrating, we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{ABC}\,, \qquad (12)$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{ABC}\,, \qquad (13)$$

$$\frac{\dot{A}}{A}-\frac{\dot{C}}{C}=\frac{k_1+k_2}{ABC}\,. \qquad (14)$$

where k_1 and k_2 are constant of integrations.

We define the average scale factor S by

$$S^3=ABC\,.$$

From equations (12)-(14), we obtain

$$\frac{\dot{A}}{A}=\frac{\dot{S}}{S}+\frac{2k_1+k_2}{3S^3}\,, \qquad (15)$$

$$\frac{\dot{B}}{B}=\frac{\dot{S}}{S}+\frac{k_2-k_1}{3S^3}\,, \qquad (16)$$

$$\frac{\dot{C}}{C}=\frac{\dot{S}}{S}-\frac{k_1+2k_2}{3S^3}\,. \qquad (17)$$

Integrating it, we get

$$A=m_1S\,\exp\left[\left(\frac{2k_1+k_2}{3}\right)\int\frac{dt}{S^3}\right], \qquad (18)$$

$$A=m_2S\,\exp\left[\left(\frac{k_2-k_1}{3}\right)\int\frac{dt}{S^3}\right], \qquad (19)$$

$$A=m_3S\,\exp\left[-\left(\frac{k_1+2k_2}{3}\right)\int\frac{dt}{S^3}\right]. \qquad (20)$$

Where m_1, m_2, m_3 are constant of integration .

We introduce volume expansion θ and σ as usual

$$\theta=v_i^i \qquad \sigma^2=\frac{1}{2}\left(\sigma_{ij},\sigma^{ij}\right),$$

σ_{ij} being shear tensor.

In the above the semicolon stands for covariant differentiation. For the Bianchi type-I metric expression for the dynamical scalar come out to be

$$\theta=3\frac{\dot{S}}{S}\,, \qquad (21)$$

$$\sigma = \frac{k}{\sqrt{3}S^3}. \quad (22)$$

Here $k^2 = k_1^2 + k_2^2 + k_3^2$ In analogy with FRW universe, we define a generalized, Hubble parameter H and the generalized deceleration parameter q as

$$H = \frac{\dot{S}}{S}, \quad (23)$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 - \frac{\dot{H}}{H^2}. \quad (24)$$

Equations (7) – (10) can be written in terms of R, σ and q as

$$\bar{p} = H^2(2q-1) - \sigma^2 + \Lambda, \quad (25)$$

$$\rho = 3H^2 - \sigma^2 - \Lambda. \quad (26)$$

From (26), we observe that $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Therefore a positive Λ restricts the upper limit of anisotropy whereas a negative more room the anisotropy.

3. SOLUTION AND DISCUSSION

The equations (4), (7)-(10) are five equations involving six unknowns term A, B, C, p, ρ, ξ and Λ so in order to close the system, we need two extra condition. We assume in the following section by formulating a special law of variation for Hubble's parameter.

We propose that the law to be examined for the variation of Hubble's parameters which yields a constant value of DP in anisotropic Bianchi I space-time is,

$$H = aS^{-m} \quad (27)$$

where a>0 and m≥0 are constants.

yields

$$S = [ma(t+t_1)]^{1/m} \quad \text{for } m \neq 0, \quad (28)$$

$$S = e^{a(t-t_0)} \quad \text{for } m = 0. \quad (29)$$

Here t_1 and t_0 are constants of integration.

Using (27) in (29), we get

$$q = m-1.$$

(30)

This shows that the deceleration parameter is constant for model (1). It may be pointed that the above law refers to anisotropic Bianchi-I space time.

Using (28) in (18)-(20), we obtain the following expression for scale factors

$$A = m_1\left[ma(t+t_1)\right]^{1/m} \exp\left[\frac{2k_1+k_2}{3a(m-3)}\left[ma(t+t_1)\right]^{m-3/m}\right], \tag{31}$$

$$B = m_2\left[ma(t+t_1)\right]^{1/m} \exp\left[\frac{k_2-k_1}{3a(m-3)}\left[ma(t+t_1)\right]^{m-3/m}\right], \tag{32}$$

$$C = m_3\left[ma(t+t_1)\right]^{1/m} \exp\left[\frac{-k_1-2k_2}{3a(m-3)}\left[ma(t+t_1)\right]^{m-3/m}\right]. \tag{33}$$

By the transformation $t+t_1=T$, $m_1x=X$, $m_2y=Y$, $m_3=Z$, the metric (1) reduces

$$ds^2 = -dT^2 + (maT)^{2/3}\left\{\exp 2\left[\frac{2k_1+k_2}{3a(m-3)}(maT)^{m-3/m}\right]dX^2 + \exp 2\left[\frac{k_2-k_1}{3a(m-3)}(maT)^{m-3/m}\right]\right.$$

$$\left. dY^2 + \exp 2\left[\frac{-k_1-k_2}{3a(m-3)}\left[ma(t+t_1)\right]^{m-3/m}\right]dZ^2\right\}. \tag{34}$$

For the model of (35), energy density ρ, cosmological term Λ are given by

$$\rho = \frac{1}{(1+\omega)}\left[2(m-2)(mT)^{-2} - \frac{2}{3}k^2(maT)^{-6/m} + 3\left\{\xi_0(mT)^{-1} + (\xi_1-\xi_2 m+\xi_2)(mT)^{-2}\right\}\right], \tag{35}$$

$$\Lambda = \left[\left(\frac{7+3\omega-2m}{1+\omega}\right)(mT)^{-2} + \left(\frac{1-\omega}{1+\omega}\right)\frac{k^2}{3}(maT)^{-6/m} - \frac{3}{(1+\omega)}\left\{\xi_0(mT)^{-1} + (\xi_1-\xi_2 m+\xi_2)(mT)^{-2}\right\}\right]. \tag{36}$$

Components of Hubble's parameter H_1, H_2, H_3, expansion scalar θ, anisotropic parameter $\bar{A}$, shear scalar σ^2 and bulk viscosity ξ are given by

$$H_1 = a(maT)^{-2} + \left(\frac{2k_1+k_2}{3}\right)(maT)^{-3/m}, \tag{37}$$

$$H_2 = a(maT)^{-2} + \left(\frac{k_2-k_1}{3}\right)(maT)^{-3/m}, \tag{38}$$

$$H_3 = a(maT)^{-2} - \left(\frac{k_1+2k_2}{3}\right)(maT)^{-3/m}, \tag{39}$$

$$\theta = 3a(maT)^{-1}, \tag{40}$$

$$\bar{A} = \frac{2k^2}{9a^2}(maT)^{\frac{2m-6}{m}}, \tag{41}$$

$$\sigma^2 = \frac{k^2}{3}(maT)^{\frac{-6}{m}}, \tag{42}$$

$$\xi = \xi_0 + (\xi_1 - m\xi_2 + \xi_2)(mT)^{-1}.$$

For the model (34), we observed that the spatial volume is zero at T=0 and expansion scalar is infinite at $T=\infty$, which shows that the universe starts evolving with zero volume at T=0 with an infinite rate of expansion. The scale factors also vanish at T=0 and hence the model has a point singularity at the initial epoch. The pressure, energy density, Hubble's factor, cosmological term and shear scalar diverges at the initial epoch provided m<3. The universe exhibits the power law expansion after the big bang impulse. As T increases the scale factors and spatial volume increase but the expansion slows down with increase in time. Also p,ρ , Λ, H_1, H_2, H_3, ξ and σ^2 deceases as T increases. As $T\to\infty$ scale factors and volume become infinite whereas p,ρ, Λ, H_1, H_2, H_3 ,θ, $\bar{A}$ and σ tend to zero but $\xi = \xi_0$. Therefore the model would essentially give an empty universe for large time T. The ratio σ/θ tends to zero as $T\to\infty$ provided m<3. Therefore the model approaches isotropy for large values of T. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.

Again using (29) in (18)-(20), we get the following expressions for scale factors.

$$A = m_1 \exp\left[a(t - t_0) - \left(\frac{2k_1 + k_2}{3a} \right) e^{-3a(t - t_0)} \right], \tag{43}$$

$$B = m_2 \exp\left[a(t - t_0) - \left(\frac{k_1 - k_3}{3a} \right) e^{-3a(t - t_0)} \right], \tag{44}$$

$$C = m_3 \exp\left[a(t - t_0) + \left(\frac{k_1 + 2k_2}{3a} \right) e^{-3a(t - t_0)} \right]. \tag{45}$$

Again by the transformation $t\text{-}t_0=T$, $m_1x=X$, $m_2y=T$, $m_3z=Z$ the metric (1) reduces to

$$ds^2 = -dT^2 + \exp(2aT)\left\{ \exp\left[-2\left(\frac{2k_1 + k_2}{3a} \right) \right] e^{-3aT} dX^2 + \exp\left[-2\left(\frac{k_2 - k_3}{3a} \right) \right] e^{-3aT} dY^2 + \right.$$

$$. \qquad \left. \exp\left[-2\left(\frac{k_1 + 2k_2}{3a} \right) \right] e^{-3aT} dZ^2 \right\}. \tag{46}$$

For the model (46) energy density ρ , cosmological term Λ are given by

$$\rho = \frac{1}{1+\omega}\left[2a^2(m-2) - \frac{2}{3}k^2 e^{-6aT} + 3a\{\xi_0 + (\xi_1 - m\xi_2 + \zeta_2)a\} \right], \tag{48}$$

$$\Lambda = \left[\left(\frac{7 + 3\omega - 2m}{1+\omega} \right) a^2 + \frac{1}{3}\left(\frac{1-\omega}{1+\omega} \right) k^2 e^{-6aT} - \frac{3a}{1+\omega}\{\xi_0 + (\xi_1 - m\xi_2 + \xi_2)a\} \right]. \tag{49}$$

Components of Hubble's parameter H_1, H_2, H_3 , expansion scalar θ, anisotropic parameter $\bar{A}$, shear scalar σ^2 and bulk viscosity ξ are given by

$$H_1 = a + \left(\frac{2k_1 + k_2}{3} \right) e^{-3aT} , \tag{50}$$

$$H_2 = a + \left(\frac{k_2 - k_1}{3} \right) e^{-3aT}, \tag{51}$$

$$H_3 = a - \left(\frac{k_1 + 2k_2}{3} \right) e^{-3aT}, \tag{52}$$

$$\theta = 3a, \tag{53}$$

$$\bar{A} = \frac{2k^2}{9a^2} e^{-6aT}, \tag{54}$$

$$\sigma^2 = \frac{k^2}{3} e^{-6aT}, \tag{55}$$

$$\xi = \xi_0 + (\xi_1 - m\xi_2 + \xi_2)a. \tag{56}$$

The model (46) has no initial singularity. The spatial volume, scale factors, pressure, energy density, cosmological term and the other cosmological parameters are constant at *T*=0. Thus the universe starts evolving with a constant volume and expands with exponential rate. As *T* increases, the scale factors and the spatial volume increase exponentially while the pressure, energy density, cosmological term, anisotropy parameter and shear scalar decrease. It is interesting to note that the expansion scalar and bulk viscosity are constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in the model. As $T\rightarrow\infty$ the scale factors and volume of the universe become infinitely large whereas anisotropy parameter and shear scalar tend to zero cosmological term Λ becomes constant for large volume of *T*. For stiff matter $(\omega = 1)$, cosmological Term Λ becomes constant throughout the evolution.

4. CONCLUSION

The solution for Bianchi type-I universe with bulk viscous fluid and cosmological Term Λ has been obtained in quadrature form. The particular case has been studied in detail. The solutions for constant deceleration parameter have been discussed in both power-law and exponential forms. Presence of bulk viscosity increases the matter density but the vacuum energy density decreases. Some of the models tend to be isotropic for large t.

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