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BIANCHI TYPE –I BULK VISCOSITY WITH CONSTANT DECELERATION PARAMETER

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ABSTRACT:
Einstein field equation with variable cosmological term Λ are cosiderred on the presence of viscous fluid for Bianchi type–I cosmological models by assuming that deceleration parameter is constant. We found that cosmological term Λ is a decreasing function of time. Physical and kinematical behaviour has been also discussed.

KEY WORDS: *Bianchi -I, Bulk viscosity, Bianchi type-I, deceleration parameter.*

1. INTRODUCTION

In the area of bulk viscous cosmology, Kalligas [1], Wesson [2], and Everitt [3] Beesham [4] Arbab [5] and Beesham have investigated FRW model as well as anisotropic Bianchi type-I cosmological models with variable gravitational constant G and cosmological constant Λ . Around the same time Arbab has considered a viscous cosmological model with variable G, Λ , and shown that the universe become isotropic in the course of expansion. We wish here to derive some results in Bianchi type-I cosmology with viscous fluid where G and Λ vary with time using slightly different method from that of Arbab.

The cosmological constant Λ is considered as one of the most important unsolved problems in cosmology. In general relativity, the cosmological constant was clearly established as a universal constant. The cosmological constant (Λ) was introduced by Einstein in 1917 as the universal repulsion to make the Universe static in accordance with generally accepted picture of that time but a general expansion of the universe was observed by Hubble in 1927 subsequently. Recent observations strongly favour a small and positive value of effective cosmological constant at the present epoch. Recent observations strongly favour a significant and a positive value of effective cosmological constant Λ with magnitude Λ(G~/c3) ≈ 10−123. Riess et al. [6-9] have recently presented an analysis of 156 SNe including a few at z > 1.3 from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. They conclude to the evidence for present acceleration q0 < 0 (q0 ≈ −0.7). Observations of Type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating.

In recent years the introduction of viscosity in cosmic fluid content has been found useful in explaining many important physical aspects of the dynamics of homogeneous cosmological models. The dissipative mechanisms not only modify the nature of singularity, usually occurring for a perfect fluid, but also can successfully account for the large entropy per baryon in the present universe. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. The physical process such as decoupling of neutrinos during the radiation era and the recombination era decay of massive super string modes into massless modes and gravitational string production. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models. See Grøn for a review on cosmological models with bulk viscosity. The first effort to create a theory of relativistic dissipative fluids, were made by Esckart and Landau & Lifshitz [10]. Weinberg established general expression for bulk and shear viscosity, and used them to evaluate the cosmological entropy production rate. A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [11]. Recently Pradhan et al. [12-19] and Singh et al. [20] have studied viscous fluid cosmological models in Bianchi type-I, V I0 and III space-times in different context.

2. MODEL AND FIELD EQUATIONS

We consider the Bianchi type-I space –time in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (1)$$

We assume that the matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (p + \bar{p})u_i v_j + \bar{p} g_{ij} \quad (2)$$

where$\bar{p} = p - \xi\theta$ (3)

We assume the matter content obey an equation of state

$$p = \omega\rho, 0 \leq \omega \leq 1 \quad (4)$$

Here , p ξ and θ are the energy density , isotropic pressure , bulk viscous coefficient and expansion scalar respectively. The flow vector v^i satisfies the condition

$$v_i v^i = -1 \quad (5)$$

The Einstein field equation with time dependent Λ , given by are

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda g_i^j \quad (6)$$

Then field equations are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + \Lambda \quad (7)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + \Lambda \quad (8)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\bar{p}+\Lambda \tag{9}$$

$$\dot{p}+(p+\bar{p})\left(\frac{\dot{A}}{AC}+\frac{\dot{B}}{CB}+\frac{\dot{C}}{AB}\right)+\dot{\Lambda}=0 \tag{10}$$

From the equations (7)– (10) we get

$$\dot{p}+(p+\bar{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\Lambda}=0 \tag{11}$$

From equation (7)-(9),we get

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{ABC} \tag{12}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{ABC} \tag{13}$$

$$\frac{\dot{A}}{A}-\frac{\dot{C}}{C}=\frac{k_1+k_2}{ABC} \tag{14}$$

k_1 and k_2 being constant of integration

We define the average scale factor by

$$R^3=ABC \tag{15}$$

From equation (12)-(14),we get

$$\frac{\dot{A}}{A}=\frac{\dot{R}}{R}+\frac{2k_1+k_2}{3R^3} \tag{16}$$

$$\frac{\dot{B}}{B}=\frac{\dot{R}}{R}+\frac{k_2-k_1}{3R^3} \tag{17}$$

$$\frac{\dot{C}}{C}=\frac{\dot{R}}{R}-\frac{k_1+2k_2}{3R^3} \tag{18}$$

Integrating it, we get

$$A=m_1R\,exp\left[\left(\frac{2k_1+k_2}{3}\right)\int\frac{dt}{R^3}\right] \tag{19}$$

$$B=m_2Rexp\left[\left(\frac{k_1-k_2}{3}\right)\int\frac{dt}{R^3}\right] \tag{20}$$

$$C=m_3Rexp\left[-\left(\frac{k_1+2k_2}{3}\right)\int\frac{dt}{R^3}\right] \tag{21}$$

Where m_1, m_2, m_3 are constant of integration we introduce volume expansionθ and σ as usual

$$\theta=v_j^i$$

$$\sigma^2 = \frac{1}{2}\left[\sigma_{ij}\sigma^{ij}\right] \tag{22}$$

σ_{ij} being shear tensor

In the above the semicolon stands for covariant differentiation. For the Bianchi type –I metric expression for dynamical scalar come out to be

$$\theta = 3\frac{\dot{R}}{R} \tag{23}$$

$$\sigma = \frac{k}{\sqrt{3}R^3} \tag{24}$$

Where $k^2 = K^2{}_1 + k^2{}_2 + k^2{}_3$

We define a generalized, Hubble Parameter H and Generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 - \frac{\dot{H}}{H^2}$$

Equation (7) –(10) can be written in terms of R ,σ and q as

$$\bar{p} = H^2(2q-1) - \sigma^2 + \Lambda$$

$$\rho = 3H^2 - \sigma^2 - \Lambda$$

3. SOLUTION AND DISCUSSION

The equations (7) -(10) and (4) are five equation involving seven unknown term A,B,C , p, ρ, Λ, ζ so in order to close the system , we need two extra condition .

We assume the form of deceleration parameter q given by

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = (constant) \tag{25}$$

Fields $R = [R(t+t_0)]^{\frac{1}{1+n}} = (R_0 T)^{\frac{1}{1+n}}$ (26)

Where R_0 and t_0 are constant of integration $T = t_0 + t$

Coefficient of bulk viscosity, ζ in the form

$$\zeta = \frac{1}{\zeta_0 + t} \tag{27}$$

Using (30) in (17)-(18), we get following expression for the scale factor

$$A = m_1 (R_0 T)^{\frac{1}{1+n}} exp\left[\frac{2k_1 + k_2}{3}\left(\frac{n+1}{n-2}\right)(R_0 T)^{\frac{n-2}{n+1}}\right] \tag{28}$$

$$B = m_2(R_0T)^{\frac{1}{1+n}} exp\left[\frac{k_2 - k_1}{3}\left(\frac{n+1}{n-2}\right)(R_0T)^{\frac{n-2}{n+1}}\right] \quad (29)$$

$$C = m_3(R_0T)^{\frac{1}{1+n}} exp\left[\frac{-k_1 - 2k_2}{3}\left(\frac{n+1}{n-2}\right)(R_0T)^{\frac{n-2}{n+1}}\right] \quad (30)$$

By the transformation, $T = t_0 + t, m_1x = X, m_2y = Y, m_3 = z$ the matric (1) reduces

$$ds^2 = -dT^2 + (R_0T)^{\frac{2}{1+n}} exap\, 2\left[\frac{2k_1 + k_2}{3}\left(\frac{n+1}{n-2}\right)(R_0T)^{\frac{n-2}{n+1}}\right]dx + (R_0T)^{\frac{2}{1+n}} \exp 2\left[\frac{k_2 - k_1}{3}\left(\frac{n+1}{n-2}\right)(R_0T)^{\frac{n-2}{n+1}}\right]dy + \exp\ 2\left[\frac{-k_1 - 2k_2}{3}\left(\frac{n+1}{n-2}\right)(R_0T)^{\frac{n-2}{n+1}}\right]\ dz \quad (31)$$

For the model (35) , expansion scalar θ, special volume v, shear scalar σ

$$\theta = \frac{3}{(1+n)^T} \quad (32)$$

$$V = (a_0T)^{\frac{3}{1+n}} \quad (33)$$

$$\sigma = \frac{k}{\sqrt{3}(R_0T)^{\frac{3}{1+n}}} \quad (34)$$

Matter density ρ, cosmological term Λ for the model take the form

$$(1+\omega)\,\rho = \frac{2(\mathrm{n}-1)}{(1+\mathrm{n})^2\mathrm{T}^2} - \frac{2\mathrm{R}^2}{3(\mathrm{R}_0\mathrm{T})^{\frac{6}{(1+\mathrm{n})}}} + \frac{3}{(\zeta_0 + \mathrm{t})(1+\mathrm{n})\mathrm{T}}$$

$$\Lambda = \frac{3\omega - 2n + 5}{(1+\omega)(1+\mathrm{n})^2(\mathrm{T})^2} - \frac{1-\omega}{1+\omega}\frac{\mathrm{R}^2}{3(\mathrm{R}_0\mathrm{T})^{\frac{6}{1+\mathrm{n}}}} - \frac{1}{(,\zeta_0 + \mathrm{t})(1+\mathrm{n})\mathrm{T}}$$

The model (35), spatial volume V is zero at T=0 where as expansion scalar θ in infinite, which shows that the universe starts evolving with zero volume at T=0 with an infinite rate of expansion. The sacle factor also vanishes at T=0 and hence the model has point type singularity at initial epoch. When T→ 0 *the* $\sigma \to \infty$, ρ→ ∞, Λ → ∞, ζ→ $\frac{1}{\zeta_0}$.

4. CONCLUSION

In this paper, we have investigated spatially homogeneous and anisotropic Bianchi type V space-time with bulk viscous matter and time varying cosmological term Λ in general relativity. The field equations have been solved exactly by using constant deceleration parameter. The universe model has been obtained and physical behaviour of the models is discussed. Coefficient of bulk viscosity ζ is assumed to be a function of time (i.e. $\zeta = \frac{1}{\zeta_0 + t}$, where ζ0 is constant). We observe that the presence of bulk viscosity increases the value of matter density. For the models obtained $\sigma/\theta \to 0$ as $T \to \infty$. Thus the models approach isotropy at late times.

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