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A COMMON FIXED POINT THEOREM USING COMMON E.A. LIKE PROPERTY IN FUZZY METRIC SPACE

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ABSTRACT:

The aim of the present paper is to prove a common fixed point theorem for four mappings satisfying common E.A. like property in fuzzy metric space, which generalize and improve the result of Rao and Kumar.

KEYWORDS: *Fuzzy metric spaces, weakly compatible mapping, common E.A. like property.*
AMS subject classification: 47H10, 54H25.

1. INTRODUCTION

In 1975, Kramosil and Michalek [1] introduced the concept of fuzzy metric space using the concept of [2], which opened an avenue for further development of analysis in such spaces.George and Veeramani [3] modified the notion of fuzzy metric spaces with the help of t-norm in 1994.

Pant [4] introduced the notion of reciprocal continuity of mappings in metric spaces. Bouhadjera and Thobie prove fixed point theorem for owc maps and weakened the concept of occasionally weak compatibility and reciprocal continuity in the form of sub compatibility and sub sequential continuity respectively and proved some interesting results with these concepts in metric spaces.

On the other hand, Aamri and El. Moutawakil [5] generalized the concepts of non-compatibility by defining the notion of E.A. property in metric space. Pant and Pant [6] defined the same property in fuzzy metric space. Wadhwa et al. [7] introduced the notion of common E.A. like property and proved some common fixed point theorems in fuzzy metric spaces. Recent result using this property can be seen in [8]. Recently, Rao and Kumar [9] proved a common fixed point theorem in a fuzzy metric space using sub compatible and sub sequentially continuous maps. In this paper we prove a common fixed point theorem for four mappings satisfying common E.A. like property in fuzzy metric spaces, which generalize and improve the result of Rao and Kumar [9].

2. PRELIMINARIES

In this section, we have recalled some definitions and useful results which are already in the literature.

Definition 2.1 [10]: A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0,1]$.

Definition 2.2 [3]: The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: $\forall\, x, y, z \in X$ and $t, s > 0$,

(FM - 1) $M(x,y,t) > 0$, for all $t > 0$,

(FM - 2) $M(x,y,t) = 1$ if and only if $x = y$,

(FM - 3) $M(x,y,t) = M(y,x,t)$,

(FM - 4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(FM - 5) $M(x,y,\cdot): [0,\infty) \rightarrow [0,1]$ is continuous.

Example: Let (X,d)be a metric space and define $a * b = min\{a,b\}$, $\forall\, x, y \in X$and $\forall t > 0$, $M(x,y,t) = \frac{t}{t+d(x,y)}$.Then$(X,M,*)$is a fuzzy metric space and the fuzzy metric M induced by the metric$d$ is often referred to as the standard fuzzy metric.

Throughout the paper $(X,M,*)$is considered to be a fuzzy metric space with condition

(FM - 6) $\lim_{t\rightarrow\infty} M(x,y,t) = 1$, for all $x, y \in X$.

Definition 2.3 [9]: Two self-maps A and Son a fuzzy metric space are said to besub sequentially continuous, iff there exists a sequence x_n in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x, x \in X$$

and satisfy

$$\lim_{n\to\infty} Ax_n = Ax, \qquad \lim_{n\to\infty} Sx_n = Sx.$$

Definition 2.4 [9]: Two self-maps A and Son a fuzzy metric space $(X, M,*)$ is are said to besub compatibleiffthere exists a sequence$x_n$in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z , z \in X$$

and which satisfy

$$lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1 , \forall t > 0.$$

Definition 2.5 [5]: Let f and g be two self-maps of a fuzzy metric space (X, M, *) then they are said to satisfy E.A property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \text{ for some } z \in X.$$

Definition 2.6 [7]: Let A, B, S and T be self maps of a fuzzy metric space (X, M, *), then the pairs (A, S) and (B, T) said to satisfy common E.A. like property if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z,$$

where$z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.

Rao and Kumar [8] proved following result:

Theorem 2.7 [9]: Let A, B, S and T be four self mappings of a fuzzy metric space (X, M, *). If the pairs (A, S) and (B, T) are sub compatible and sub sequentially continuous, then
(I) and have a point of coincidence,
(II) and have a point of coincidence.

Further, if s

(III) $\emptyset(min\{M(Ax,By,t)M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t)\}) \geq 0$

For all x, y∈X and t >0, where $\emptyset : [0,1] \rightarrow [0,1]$ is a continuous function with $\emptyset(s) > s$, for each$0 < s < 1$. Then A,, and T have a unique common fixed point in X.

We use the following in our results motivated by [11]:

(α^*)Let Φbe the set of all real valued continuous function $F : [0,1]^4 \rightarrow [0,1]$suchthat

$(\alpha_{2.1})F$is non increasing in 2nd, 3rd and 4thcoordinate variables and

$(\alpha_{2.2})$if$F(v,1,v,1) \geq 0, F(v,1,1,v) \geq 0, F(v,v,1,1) \geq 0$,

for all $v \in [0,1]$ implies $v = 1$.

3. MAIN RESULT

Theorem: Let A, B, S and T be selfmappings of a fuzzy metric space$(X,M,*)$ satisfying the following conditions:

(3.1.1) for all $x,y \in X$and$t > 0$,

$$F\left(\begin{matrix} M(Ax,By,t), \dfrac{\alpha_2 M(Ty,Ax,t) + \alpha_1 M(Ty,Sx,t)}{\alpha_1 + \alpha_2 M(Ax,Sx,t)}, \\ M(Ax,Sx,t), M(By,T\,y,t) \end{matrix}\right) \geq 0,$$

Where$F \in \Phi$ and $\alpha_1, \alpha_2 \geq 0$, which cannot be simultaneously 0,

(3.1.2) pairs$(A,S)$and$(B,T)$is weakly compatible mappings and enjoys the common E.A. like property, then A, B, S and T has a unique common fixed point in X.

Proof: Since (A, S) and (B, T) satisfy common E.A. like property therefore there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z_1$$

where $z_1 \in S(X) \cap T(X)$ or $z_1 \in A(X) \cap B(X)$.

Suppose $z_1 \in S(X) \cap T(X)$, now we have $\lim_{n\to\infty} Ax_n = z_1 \in S(X)$ then z_1= Su for some u ∈X.

No we claim that Au = Su, from (3.1.1) we have,

$$F\begin{pmatrix} M(Au, By_n, t), \dfrac{\alpha_2 M(Ty_n, Au, t) + \alpha_1 M(Ty_n, Su, t)}{\alpha_1 + \alpha_2 M(Au, Su, t)}, \\ M(Au, Su, t), M(By_n, Ty_n, t) \end{pmatrix} \geq 0,$$

Taking limit $n \to \infty$, we get

$$F\begin{pmatrix} M(Au, z_1, t), \dfrac{\alpha_2 M(z_1, Au, t) + \alpha_1 M(z_1, z_1, t)}{\alpha_1 + \alpha_2 M(Au, z_1, t)}, \\ M(Au, z_1, t), M(z_1, z_1, t) \end{pmatrix} \geq 0,$$

$$F(M(Au, z_1, t), 1, M(Au, z_1, t), 1) \geq 0.$$

Using(α^*), we have, $M(Au, z_1, t) = 1$. Hence Au $= z_1 =$ Su.

Since the pair (A, S) is weak compatible, therefore Az_1= ASu= SAu= Sz_1.

Again, $\lim_{n\to\infty} By_n = z_1 \in T(X)$ then z_1= Tvfor some v∈X.

No we claim that Tv= Bv, from (3.1.1) we have,

$$F\begin{pmatrix} M(Ax_n, Bv, t), \dfrac{\alpha_2 M(Tv, Ax_n, t) + \alpha_1 M(Tv, Sx_n, t)}{\alpha_1 + \alpha_2 M(Ax_n, Sx_n, t)}, \\ M(Ax_n, Sx_n, t), M(Bv, T\,v, t) \end{pmatrix} \geq 0,$$

Taking limit $n \to \infty$, we get

$$F\begin{pmatrix} M(z_1, Bv, t), \dfrac{\alpha_2 M(z_1, z_1, t) + \alpha_1 M(z_1, z_1, t)}{\alpha_1 + \alpha_2 M(z_1, z_1, t)}, \\ M(z_1, z_1, t), M(Bv, z_1, t) \end{pmatrix} \geq 0,$$

$$F\big(M(z_1, Bv, t), 1,1, M(Bv, z_1, t)\big) \geq 0,$$

Using(α^*), we have, $M(Bv, z_1, t) = 1$. Hence Bv $= z_1 =$ Tv.

Since the pair (B, T) is weak compatible, therefore Tz_1= TBv= BTv= Bz_1.

Now we show that Az_1= z_1, from (3.1.1) we have,

$$F\begin{pmatrix} M(Az_1, By_n, t), \dfrac{\alpha_2 M(Ty_n, Az_1, t) + \alpha_1 M(Ty_n, Sz_1, t)}{\alpha_1 + \alpha_2 M(Az_1, Sz_1, t)}, \\ M(Az_1, Sz_1, t), M(By_n, Ty_n, t) \end{pmatrix} \geq 0,$$

Taking limit $n \to \infty$, we get

$$F\begin{pmatrix} M(Az_1, z_1, t), \dfrac{\alpha_2 M(z_1, Az_1, t) + \alpha_1 M(z_1, Az_1, t)}{\alpha_1 + \alpha_2 M(Az_1, Az_1, t)}, \\ M(Az_1, Az_1, t), M(z_1, z_1, t) \end{pmatrix} \geq 0,$$

$$F(M(Az_1, z_1, t), M(z_1, Az_1, t), 1, 1) \geq 0,$$

Using(α^*), we have, $M(Az_1, z_1, t) = 1$. Hence $Az_1 = z_1$.

Now we show that $Bz_1 = z_1$, from (3.1.1) we have,

$$F\begin{pmatrix} M(Ax_n, Bz_1, t), \dfrac{\alpha_2 M(Tz_1, Ax_n, t) + \alpha_1 M(Tz_1, Sx_n, t)}{\alpha_1 + \alpha_2 M(Ax_n, Sx_n, t)}, \\ M(Ax_n, Sx_n, t), M(Bz_1, Tz_1, t) \end{pmatrix} \geq 0,$$

Taking limit $n \to \infty$, we get

$$F\begin{pmatrix} M(z_1, Bz_1, t), \dfrac{\alpha_2 M(Bz_1, z_1, t) + \alpha_1 M(Bz_1, z_1, t)}{\alpha_1 + \alpha_2 M(z_1, z_1, t)}, \\ M(z_1, z_1, t), M(Bz_1, Bz_1, t) \end{pmatrix} \geq 0,$$

$$F(M(z_1, Bz_1, t), M(Bz_1, z_1, t), 1, 1) \geq 0,$$

Using(α^*), we have, $M(Bz_1, z_1, t) = 1$. Hence $Bz_1 = z_1$.

Hence, $Az_1 = Sz_1 = Bz_1 = Tz_1 = z_1$.Thus $z_1$is common fixed point of A, B, S and T.

To prove uniqueness we suppose that p and q are two common fixed point of A, B, S and T such that $p \neq q$, then from (3.1.1) we have,

$$F\begin{pmatrix} M(Ap, Bq, t), \dfrac{\alpha_2 M(Tq, Ap, t) + \alpha_1 M(Tq, Sp, t)}{\alpha_1 + \alpha_2 M(Ap, Sp, t)}, \\ M(Ap, Sp, t), M(Bq, Tq, t) \end{pmatrix} \geq 0,$$

$$F\begin{pmatrix} M(p, q, t), \dfrac{\alpha_2 M(q, p, t) + \alpha_1 M(q, p, t)}{\alpha_1 + \alpha_2 M(p, p, t)}, \\ M(p, p, t), M(q, q, t) \end{pmatrix} \geq 0,$$

$$F(M(p, q, t), M(q, p, t), 1,1) \geq 0,$$

Using(α^*), we have, $M(p, q, t) = 1$. Hence $p = q$. This completes the proof of the theorem.

4. CONCLUSION:

We proved a common fixed point theorem for common E.A. like property in fuzzy metric space and generalized and improve the result of Rao and Kumar. In our result we do not require sub compatible and sub sequentially continuity to obtained common fixed point.

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